ELEN 3401 Electromagnetics
Problem Set #5

DUE: Friday March 7
Please include your name and UNI on the assignment

Problem 1: Conservative vector fields

The work done by a force to move an object from point P0 to P1 is defined as

$$W = \int_{P0}^{P1} \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{dl}}$$

a. If the work done by the electric field to move a charge q between two points is independent of the path that the charge takes, show that:

$$\oint_C \vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{dl}} = 0$$

where C can be any closed path. Use the Stokes theorem to show that this is equivalent to $\nabla \times \boldsymbol{E} = 0$

b. Find a, b, c such that $\vec{\boldsymbol{E}} = (x + 2y + az)\,\hat{x} + (bx - 3y - z)\,\hat{y} + (4x + cy + 2z)\,\hat{z}$ is a conservative field, i.e. $\vec{\nabla} \times \vec{\boldsymbol{E}} = 0$

c. Find its scalar potential $V(x, y, z)$ such that: $\vec{\boldsymbol{E}} = -\vec{\nabla} V$

d. Obtain $\vec{\nabla} \cdot \vec{\boldsymbol{E}}$

Problem 2: Parallel plate capacitor

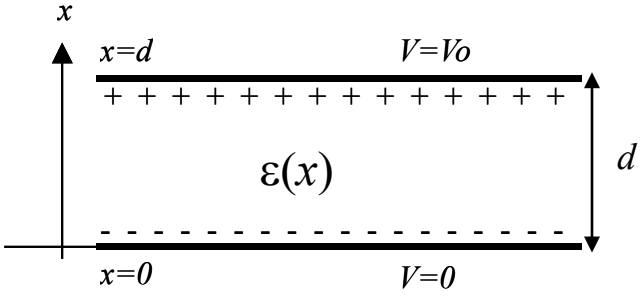
Consider the parallel plate capacitor in the figure below. The region between the plates is filled with a perfect dielectric of non-uniform permittivity, *ε(x):*

$$\epsilon(x) = \frac{\varepsilon_o}{1 - \frac{x}{2d}}$$

The electric potential, *V(x),* at *x=d* is V_o, and its reference is taken at *x=0*, such that *V(x=0) = 0.*

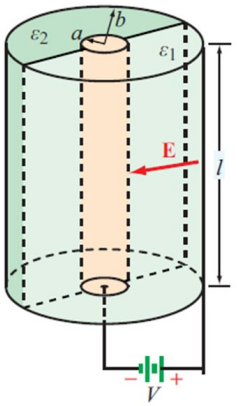
a. Obtain the electric potential, *V(x),* for the region between the plates (*0<x<d*).

b. Obtain an expression for the capacitance per unit area.



Problem 3: Coaxial capacitor

A coaxial capacitor consists of two concentric, conducting, cylindrical surfaces, one of radius a and another of radius b, as shown in the figure below. The insulating layer separating the two conducting surfaces is divided equally into two semi-cylindrical sections, one filled with dielectric ϵ_1 and the other filled with dielectric ϵ_2.



a. Develop an expression for the coax capacitance, C in terms of the length l, the geometrical parameters, and the dielectric constants.
b. Evaluate the value of C for a = 6 mm, b = 10 mm, ϵ_{1r} = 2, ϵ_{2r} = 6, and l = 8 cm.